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MASS TRANSFER BETWEEN THE GAS AND SOLID PARTICLES IN A FLUIDIZED BED

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A model of the boundary layer and condition of suspension of a solid particle by a gas flow is used to formulate analytical correlations, which are then compared with experiment.

Fluidization techniques have come increasingly under the scrutiny of power engineers in recent years in connection with the growing use of coal as a source of energy in the face of tightening environmental-protection demands.

Coal combustion takes place in a fluidized bed of coarsely disperse material, and the process is largely governed by mass transfer toward the surface of the burning particle. The mass transfer between the gas and particulate material in a fluidized bed has been studied under conditions such that all particles are involved in the transfer process. A wealth of experimental data has been accumulated to date and has been generalized in a survey paper [1]. Inasmuch as the concentration of coal in the fluidized bed is small, not more than 1-2%, the available information is unfortunately of only minor practical interest. An exception is the recently reported work of Hsiung and Thodos [2] on the sublimation of naphthalene in a fluidized bed of polymer particles. The authors have obtained some interesting experimental data, but they failed to comprehend the mass-transfer laws in the system. Our present objective is to bridge this gap and to investigate the mass transfer between the particles and the gas in a fluidized bed on the basis of reasonably general properties of the system, namely that:

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a) the particles are aerated by a laminar gas flow, and a Prandtl boundary layer is formed on their surface, governing the momentum- and mass-transfer processes;

b) the particles are suspended by the gas, and the average tangential stresses created on their surface by viscous friction are determined by the weight of the particles.

In a fluidized bed the relative velocity of flow of the gas around the particles is small and close to the velocity for the initiation of fluidization. The corresponding Reynolds numbers do not exceed 100, so that the gas in the intergranular channels moves in laminar flow [3, 4]. Individual clusters of particles, which alternately break up and reform anew, move continuously through the space of the fluidized bed. The lifetime τ_0 of such formations when the motion of adjacent particles is concurrent (correlated) is ≈ 50 msec. Low-frequency oscillations prevail in the system, and all fluctuations fall in the frequency interval below 25 Hz. These inferences are based on data from measurements of the spectrum of velocity fluctuations of the material in fluidized beds [5].

The gas passes by a particle quickly, so that its passage time is much shorter than the lifetime of the moving particle clusters, i.e., than the period of the fastest fluctuations of the relative velocity of the gas. The corresponding Strouhal number is small:

$$\mathrm{S} = \frac{\tau}{\tau_0} = \frac{d\varepsilon_0}{U_0\tau_0} = \frac{d^2\varepsilon_0}{\mathrm{Re}_0 \nu\tau_0} < 0.05.$$

From the total mass of particles moving in the fluidized bed we isolate a single "test" particle and trace the pattern of motion of the gas aerating it. At the surface of the test particle the adjacent particles are continually changing, and the gas streams in the intergranular channels change direction in random fashion. The velocity field of the gas flowing around the test particle, when averaged over the ensemble of possible realizations, appears fairly uniform and is determined by the maximum velocity of the gas in the intergranular channels of the swarm of adjacent particles. Since this velocity is approximately an order of magnitude greater than the velocity for the initiation of fluidization, the corresponding Reynolds number, which characterizes the flow of gas around a spherical test particle, will be comparatively large: $\mathrm{Re} > 10$.

The interchanging of adjacent particles in the swarm produces fluctuations of the relative gas velocity. These fluctuations have a low frequency, and the quantity

$$\mathrm{S}\,(U_a/\bar{U}_\infty) < 0.02,$$

so that, as shown by experiments with isolated bodies [6], the heat-transfer process is quasisteady. The time-average heat-transfer coefficient in this case does not depart from its values under nonfluctuating conditions. We assume that the fluctuations of the gas velocity have a normal distribution function and that their variance is close to the mean. In other words, it is anticipated that the velocity fluctuations will increase with the flow velocity of the gas around the test particle.

Thus, in the fluidized bed a steady-state boundary layer evolves around the test particle. The impingent gas flow on it is uniform on the average and is characterized by the maximum velocity of the gas in the intergranular channels of the surrounding particle swarm. The corresponding Reynolds number is large, $\mathrm{Re} > 10$, and so the process of mass and momentum transfer is described by the Prandtl boundary-layer equations.

Normally the gases used in fluidized beds have Schmidt numbers close to unity, and the mass transfer through the steady-state boundary layer formed on the test sphere can be described by means of the well-known solution of the Prandtl boundary-layer equations for a sphere in a uniform flow [3]:

$$\mathrm{Sh} = k\,\mathrm{Sc}^{1/3}\sqrt{\mathrm{Re}}\,, \tag{1}$$

where

$$k = \frac{\overline{U_\infty^{0.5}}}{(\bar{U}_\infty)^{0.5}}\,\frac{I}{2}\,.$$

It is evident from Eq. (1) that the mass transfer depends on the dimensionless velocity gradient of the gas at the surface of the test particle (I). We determine the latter quantity from the solution of the hydrodynamic problem, making use of the self-similar solution

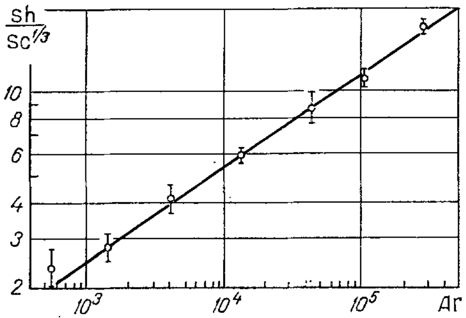

Fig. 1. Mass-transfer coefficient in a fluidized bed versus the characteristics of the material according to Eq. (10) (solid line) and the experimental data of [2] (circle-points) with standard deviations (vertical bars).

of the Prandtl boundary-layer equations [3, 4] describing the tangential stresses at the surface of a sphere in a uniform gas flow:

$$\sqrt{\frac{d_i}{\nu U_\infty^3}}\,\frac{\tau_w}{\rho} = K(x), \tag{2}$$

where $K(x) = \sum_{n=0}^{3} a_n x^{2n+1}$ and for n = 0, 1, 2, 3, respectively, a_n = 3.4086, −1.3378, 0.1434, $-8.828 \cdot 10^{-3}$.

The quantity K(x) reflects the variation of the dimensionless velocity gradient along the surface of the sphere. Integrating along the generatrix to the point of flow separation, we determine the average value of the velocity gradient on the surface of the sphere.

We now average the tangential stresses created at the surface of the particle and take two factors into consideration: 1) fluctuations of the impingent flow velocity; 2) the possible formation of stagnation zones near the points of contact of adjacent particles moving concurrently.

We first determine in general form the correction for averaging over the possible realizations of the impingent flow velocity. Expanding the power function into a series and making use of the fact that all moments higher than the second vanish, after integration we obtain

$$\eta = \frac{\overline{U_\infty^n}}{(\overline{U}_\infty)^n} = \frac{1}{(\overline{U}_\infty)^n}\int\limits_{-\infty}^{+\infty}\frac{U_\infty^n}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(U_\infty-\overline{U}_\infty)^2}{2\sigma^2}\right]dU_\infty \simeq 1+\frac{n(n-1)}{2}\left(\frac{\sigma}{\overline{U}_\infty}\right)^2. \tag{3}$$

For n = 1.5 and 0.5 this correction is equal to, respectively, η = 1.375 and 0.875. We then take the second factor into account. When the particles move concurrently, stagnation zones can form near the contact points, and part of the surface of the test particle will not participate in momentum transfer. Depending on the thickness of the hydrodynamic boundary layer and the nature of the particle motion, the cross section through which momentum is transferred will vary from 1 at the particle surface to the porosity of the swarm (ε_0 = 0.48) over a distance greater than the radius of the surrounding particles. Inasmuch as the interchanging of adjacent particles is a random process, as an estimate we take the arithmetic mean $\bar{\varepsilon} = 0.5 \cdot (\varepsilon_0 + 1) = 0.74$.

After suitable transformations we finally obtain

$$\frac{\bar{\varepsilon}}{\eta}\sqrt{\frac{d_i}{\nu(\overline{U}_\infty)^3}}\,\frac{\bar{\tau}_w}{\rho} = I, \tag{4}$$

where

$$I = \frac{1}{0.965}\int\limits_0^{0.965} K(x)\,dx = 1{,}428, \tag{5}$$

$$\eta = \frac{\overline{U_\infty^{1.5}}}{(\overline{U}_\infty)^{1.5}}. \tag{6}$$

All particles in the fluidized bed are suspended by the gas flow, and they ostensibly "float on the gas." In this case the average tangential stresses at the surface of each particle of the bed remain constant at all times and equal to its weight. The average tangential stresses on the surface of a spherical particle are

$$\bar{\tau}_w = \frac{\langle \text{particle weight} \rangle}{\langle \text{particle surface} \rangle} = g\,\frac{d_i}{6}\,(\rho_p - \rho). \tag{7}$$

Eliminating the average tangential stresses from (4) and (7) and making suitable transformations, we obtain

$$\left(\frac{d_i}{\nu}\right)^2 g\,\frac{d_i}{6}\,\frac{\rho_ч - \rho}{\rho} = \frac{I\eta}{\bar{\varepsilon}}\sqrt{\frac{\nu \bar{U}_\infty^3}{d_i}}\left(\frac{d_i}{\nu}\right)^2,$$

$$\mathrm{Ar} = 6\,\frac{I\eta}{\bar{\varepsilon}}\,\mathrm{Re}_e^{3/2}, \quad \text{or} \quad \mathrm{Re}_e = \left(\frac{\mathrm{Ar}\bar{\varepsilon}}{6I\eta}\right)^{2/3}. \tag{8}$$

Relation (8) shows that the effective velocity (Reynolds number) governing the transfer processes is a function only of the physical properties of the material suspended by the gas flow (Archimedes number). Its derivation involves the Prandtl boundary-layer approximation, which is valid for large Reynolds numbers:

$$\mathrm{Re} > 10; \quad \mathrm{Ar} = \frac{6I\eta}{\bar{\varepsilon}}\,\mathrm{Re}^{1.5} > 500. \tag{9}$$

Now, using relation (8) for the effective Reynolds number, we transform the original relation (1) characterizing the mass-transfer laws in application to the general case where the diameter of the test particle differs from the effective diameter of the particles forming the fluidized bed:

$$\mathrm{Sh} = k\,(\mathrm{Sc\,Ar})^{1/3}\,(d/d_i)^{1/2}, \tag{10}$$

where

$$k = \frac{I\eta}{2}\left(\frac{\bar{\varepsilon}}{6I\eta}\right)^{1/3} = 0.248.$$

Thus, Eq. (10) describes the mass-transfer laws in a fluidized bed between the gas and a relatively large particle migrating in a bed of smaller particles. The mass-transfer coefficient in this case does not depend on the gas filtration rate, but is determined by the average tangential stresses created at the surface of the particles suspended by the gas flow (Archimedes number).

It is instructive to compare relation (10) with the experimental data of an investigation [7, 8] of the combustion of solitary carbon particles in a fluidized bed. High air excesses were maintained in the experiments, so that the combustion process took place in the diffusion region and was determined by the oxygen supply rate. In a fluidized bed of incombustible material, solitary carbon particles with diameters of 3 to 12 mm burned at a bed temperature of 1000-1200°K. The material forming the bed consisted of narrow fractions of sand or refractory clay with grain sizes varied from 0.4 to 2.15 mm. The total combustion time of a carbon sphere having a known initial diameter was recorded in the experiments. The details of the experimental procedure are described in [7, 8]. From the experimental data we estimated the average combustion rate, which we then used to calculate the mass-transfer coefficient. Processing of the experimental results yielded a correlation similar to (10) and differing only in the value of the proportionality factor: $k = 0.24$.

Considering that the error of determination of the proportionality factor in the empirical correlation falls within 8% limits, the agreement of the experimental and theoretical relations is reasonably good. We now compare the correlation (10) with the experimental data of a study [2] of mass transfer in a fluidized bed at room temperatures, in which case a fairly high measurement accuracy is attainable. Figure 1 shows the average values of the mass-transfer coefficient and its standard deviations. The experiments were conducted on the sublimation of spherical naphthalene pellets in a fluidized bed of polymer particles with the same density and size as the naphthalene spheres. The theoretical relation (10) is plotted in the figure for conditions when the diameter of the test particle coincides with that of the inert particles. The experimental points provide a good fit to the theoretical function (10). A certain deviation of the experimental points from the line is observed only in the interval of relatively fine particles, where the Archimedes number is small ($Ar < 10^3$) and the condition (9) for applicability of the Prandtl boundary-layer model breaks down.

Thus, the described experiments on the combustion of coke particles and on the sublimation of naphthalene in fluidized beds corroborate the theoretical relation (10), which reflects the fundamental laws of mass transfer between the gas and solid particles in a fluidized bed. The agreement of the theoretical and experimental relations supports the validity of the original notions that a hydrodynamic boundary layer is formed around a particle moving in a fluidized bed, its nature being determined by the condition of suspension of the particles in the gas flow.

NOTATION

d, d_i, diameters of, respectively, a test particle and all other particles forming a fluidized bed; D, molecular diffusivity; g, acceleration of gravity; I, dimensionless average velocity gradient over the surface of a particle; U_∞, U_a, $\bar{U}_\infty$, velocity of gas impinging on the test particle, its amplitude, and its average value; U_0, velocity for the initiation of fluidization; x, coordinate along the surface of the test sphere; β, mass-transfer coefficient; η, correction for fluctuations of the gas velocity; ν, kinematic viscosity of the gas; ρ, ρ_p, densities of the gas and the particulate material; τ_w, $\bar{\tau}_w$, tangential stress at the surface of a particle and its average value; σ, standard deviation of the impingent flow velocity; $Re = U_\infty d_i/\nu$, $Sc = \nu/D$, $Sh = \beta d/D$, $S = d/U_0\tau_0$, $Ar = (gd_i^3/\nu_2)\cdot[(\rho_p - \rho)/\rho]$.

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